Civilization of S. & N. Ronart (Praeger, NY, 1960), by no means an authoritative work, and even then the claim of the author is left without any support.

The author's stated purpose in writing this book "is not simply to record isolated discoveries, but rather to explain the progress of mathematical thought" (pp. 14, 93). That he attempts to do in seven short chapters: an introduction and a conclusion, and the five remaining ones are devoted to Historical Setting, Arithmetic, Algebra, Trigonometry and Geometry. The book falls quite short of its aim, for after reading it through one is left with little more than a list of a few famous names of mathematicians and a few of their works loosely connected by very general statements. Add to that the many mistakes, a few typographical errors, and the incomplete documentation that is most frustrating to any reader who requires any clarity whatsoever; e.g., none of the manuscripts, mainly from the India Office, referred to in the text and in the bibliography carry any titles and the reader is left on his own to struggle even for the subject matter of these works.

Moreover, the book is marred with more serious unfounded statements that may lead the innocent reader to believe that the Moslem mathematicians *invented* the zero and the decimal system and *discovered* the sphericity of the earth. One reads, for example: "As a result of their studies, the Muslims established the fact that the earth is a sphere floating in space" (p. 96); also: "Muslim mathematicians invented (*sic*) the present arithmetical decimal system..." (p. 7); and "Moslems Offered the Zero" (p. 37).

With the state of our knowledge of Islamic mathematics being as it is, one can realistically hope for a short coherent book on what is known so far, and an updating of the secondary literature. But much more thorough research need to be done before the aim of this book could be reached.

Finally one ought to congratulate the designer of the dust jacket for his selection of a manuscript illustration from a work of Ibn al-Shatir of Damascus (1375 A.D.) which depicts the lunar model, itself having a great affinity to that of Copernicus and has been the subject of several papers in recent years.

A HISTORY OF NUMERICAL ANALYSIS FROM THE 16TH THROUGH THE 19TH CENTURY. By Herman H. Goldstine. (Springer). 1977.

Reviewed by Garrett Birkhoff,
Dept. of Math., Harvard Univ., Cambridge, MA 02138

The above companion piece to Dr. Goldstine's well-known *The Computer from Pascal to von Neumann* contains a valuable collection of summaries of important papers on numerical analysis by leading mathematicians. As he writes in his preface, he has "attempted to trace the development of numerical analysis during the period in which the foundations of the modern theory were being laid". He has "rather arbitrarily chosen, in the main, the most famous mathematicians ... and concentrated on their major works in numerical analysis." On the whole, Dr. Goldstine has been eminently successful in achieving his stated purpose, insofar as work up to 1840 or so is concerned.

Thus his book begins with a charming account of the construction of logarithm tables by Napier, Briggs, Burgi, and Kepler, and of the impetus that their work gave to interpolation techniques. It also includes an amusing summary of the solution by Vieta (1540-1603) of a polynomial equation of degree 45, inserted somewhat out of chronological order. After a brief sketch of some results by Descartes and "his great intellectual rival Fermat" on root-finding, there follows a scholarly account of Newton's fundamental contributions. Newton and his friend the astronomer Halley devoted much effort to computing more accurate logarithm tables. Thus Newton computed $\log_e 1.1$ to 68 decimal places, incorrectly in 1664 and correctly in 1676. Halley used the Mercator-Wallis-Gregory series

$$\log [(1 + q)/(1 - q)] = 2[q + q^3/3 + q^5/5 + \dots]$$

to compute other values accurately. Newton's ingenious methods for computing trigonometric tables are described next, followed by the Newton-Raphson "improved version" of the Vieta-Oughtred root-finding procedure, and then Newton's masterful treatment of finite differences on a uniform mesh. The chapter concludes with vivid sketches of the derivation by Maclaurin of the Euler-Maclaurin sum formula, and by Stirling of Stirling's formula.

The next long chapter on "Euler and Lagrange" marks a transition from *numerical* analysis to numerical *analysis*. Thus it begins with Euler's 125 digit value of π, his clever summation of the first 1000 natural logarithms to 18 places, and so on, as well as his (prior) derivation of the Euler-Maclaurin sum formula as a means for computing γ, and his work on interpolation. There follows synopses of Lagrange's contributions to difference equations, functional equations, to polynomial and trigonometric interpolation, and to "hidden periodicities", all written by Lagrange in a quite different (analytical and algebraic) style.

Then come synopses of seven contributions by Laplace, three from the 1770's (presented out of chronological order), the others from his *Mécanique Céleste* (1799-1805) and *Théorie des Probabilités* (first ed. 1812). They illustrate Laplace's skill in using generating functions, the operational calculus, and the Laplace transform.

There follows the "method of least squares" story, involving

Laplace, Legendre and Gauss. Legendre's important constructions of tables of elliptic integrals and of primes are ignored. Next come summaries of Gauss on quadrature, on interpolation, and on rounding errors. Especially fascinating are excerpts from Gauss' posthumously published paper on interpolation which show that he had anticipated the discovery of the Fast Fourier Transform.

The final chapter contains interesting summaries of selections from Jacobi (1826, 1834, 1845), followed by some snippets from Cauchy (1821, 1827, 1840). Cauchy's method of rational interpolation is described and its extension by Padé is mentioned by Pade (but not explained). The book then concludes with a handful of other nineteenth century results, due to Heun, Runge, Hermite, Abel, and others. Thus the reader's final impression is rather kaleidoscopic.

The stimulus to several of the preceding investigations provided by celestial mechanics and by navigation problems is illustrated by several intriguing examples (Sec. 3.6, pp. 251-2, and elsewhere). However, no systematic account is given either of these fields, or of Euler's interest in ballistics (which led him to invent techniques of numerical integration still used a century after his death). Neither is any idea given of the importance of Gauss' role as observatory director from 1807 on, nor of the immense geodetic computations which he planned and helped perform. Moreover although Adams' computation of γ to 263 places is mentioned, the calculations which led him (and Leverrier) to discover Neptune are not. Neither are the Delaunay-Whittaker approximations of lunar theory, nor Hill's fundamental methods (which so inspired Poincaré). Likewise, the calculations of tidal tables are ignored. Thus in general, the reader is not made conscious of the major contribution made by numerical methods to European scientific and industrial progress in the 19th century.

Finally, it is questionable whether "the foundations of the modern theory" were actually in fact laid in the papers so skillfully summarized by Dr. Goldstine. Although Gauss and Jacobi developed some of the more important *algorithms* for solving simultaneous linear equations, the modern theory of these algorithms is based on matrix concepts such as that of 'factoring a matrix', unknown to Gauss or Jacobi. More important, little or no space is devoted to the ideas of counting operations ("computational complexity"), of norm and function space, of the *approximation* of functions, whether orthogonal (Euler-Fourier) [See for example §382 of Fourier's *Théorie Analytique de la Chaleur*] or uniform (Chebyshev-Weierstrass), or of data fitting (by Kepler or by actuaries).

But, it is also questionable whether the preceding omissions could have been repaired without nearly doubling the size of the book. The scientific world should be very grateful to Dr.

Goldstine for his clear and careful summaries of many influential papers, accompanied by a wealth of insightful mathematical and historical remarks.

GAUSS UND DIE NICHT-EUKLIDISCHE GEOMETRIE. By Hans Reichardt. Leipzig (Teubner). 1976. 116 pp.

Reviewed by Jeremy Gray,
Faculty of Math., Open University, Milton Keynes, England

This useful little book was commissioned on the 200th anniversary of Gauss' birth to assess his significance for non-Euclidean geometry. In three chapters it surveys the work of Gauss' predecessors and that of Gauss and his contemporaries, and develops the elementary differential geometry of the non-Euclidean plane.

The development of non-Euclidean geometry was important for many reasons: it has rich mathematical consequences; it affected the subsequent direction of mathematical research by challenging and enhancing contemporary ideas about the nature of geometry; it raised questions about the mathematization of nature and the very logical consistency of mathematics itself. Several of these developments could only take place after the general acceptance of the new geometry, so, although Reichardt is well aware of them, he wisely chooses to concentrate on those matters which characterized the changing problem-situation in the period up to Gauss. These he presents through extensive quotations, 25 of the 67 pages in the chapter on Gauss and his contemporaries are taken from primary sources. In this way one can trace the development of Gauss' thought as it was revealed, chiefly in letters to his friends, from skepticism concerning the purported proofs of the parallel postulate to confidence in the validity of a new geometry. One may likewise follow the reasoning of the Bolyais and Lobachevski over the years, and incidentally gain a vivid picture of the protagonists as they struggled to create "another, new world of nothing" as Janos Bolyai put it (p. 58). It is encouraging to note that more attention is paid than is usual to Lobachevskii's early work, published in Russian in 1829 and 1835 and partly in French in *Crelle's Journal* in 1837, which amplifies his better known German book of 1840. In all of this Reichardt is able to draw upon the remarkable scholarship of Engel and Stäckel around the turn of the century in making widely available the work of Gauss, the Bolyais, Lobachevskii, and others, and in providing them with discerning commentaries. Although these books are well-known to the experts the author provides a useful service in making extracts available to the general reader, and thereby directing his attention to the original writings. In this